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An analytical procedure is presented for determining approximate elastic-plastic behavior of individual stories in an unbraced multi-story steel frame subjected to nonproportional combined loading. The procedure is based on sway subassemblages and considers the second order P-Δ effect. The original approach to this method is expanded to include column axial shortening for analysis of individual stories. This is shown to affect significantly horizontal drift of a story and is also shown to influence the order of plastic hinge formation. The use of general parameters for the assumptions of boundary conditions allows all regions of the frame to be analysed. These parameters can be conservatively chosen, thereby allowing a conservative analysis. Also, simplifying assumptions make the method easy to apply to direct tabular computation.

The individual story behavior obtained by this method has been compared with two full frame analyses, considered more exact. The comparisons show very good agreement of results, indicating that this approximate method is accurate and conservative. The results therefore indicate the method is suitable for checking frame strength and stiffness.

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ABSTRACT

An analytical procedure is presented for determining approximate elastic-plastic behavior of individual stories in an unbraced multi-story steel frame subjected to nonproportional combined loading. The procedure is based on sway subassemblages and considers the second order P-Δ effect. The original approach to this method is expanded to include column axial shortening for analysis of individual stories. This is shown to affect significantly horizontal drift of a story and is also shown to influence the order of plastic hinge formation. The use of general parameters for the assumptions of boundary conditions allows all regions of the frame to be analysed. These parameters can be conservatively chosen, thereby allowing a conservative analysis. Also, simplifying assumptions make the method easy to apply to direct tabular computation.

The individual story behavior obtained by this method has been compared with two full frame analyses, considered more exact. The comparisons show very good agreement of results, indicating that this approximate method is accurate and conservative. The results therefore indicate the method is suitable for checking frame strength and stiffness.

1. INTRODUCTION

The tall building will be a predominant structure for housing people to live and work. Plastic methods may offer economy in the design of steel buildings. Strength and stiffness criteria must be met to achieve satisfactory preformance of tall buildings, and simplified methods are needed to make certain these criteria are met. Therefore, this thesis will present extensions to simplify and to make more general a method to analyse unbraced frames called the sway subassemblage method of analysis.

The subassemblage method gives the utmost aid to the engineer to visualize and compute the load deflection behavior of bare steel multistory frames. Its conceptual contribution lies in its simplistic approach to dividing the frame into smaller units which can be more easily handled than trying to visualize the behavior of a highly redundant frame.

The use of the digital computer has immensely aided the analysis of structures; however, often with "canned" programs, important structural engineering concepts of analysis may be lost without proper scrutiny. On the other hand, the subassemblage approach allows close examination of a small part of the frame, such as one story, which manifests the use of this lucid approach to analysis. It can be applied either by hand calculation or could be done efficiently by digital computer. It achieves the non-proportional analysis of a story of a frame and accomplishes the aim of an easy means of judging frame strength and stiffness.

Because the subassemblage method relies upon the division of a frame into small parts which are easily analysed, it is noticed that the benefit of this method is the potential for redesign once criteria have been set for strength and stiffness. The potential to achieve an improved design is important to the designer who is seeking economy and safety.

1.1 Preliminary Design

An extensive elaboration on the preliminary design of unbraced frames is not presented in this thesis. Any method may be used to select preliminary beam and column sections. Driscoll[8] and Hansell[9] present plastic design procedures, using the technique of plastic moment balancing.

The moment balancing method bases the design on formulation of equilibrium and plastic girder mechanism. The procedure for the design of girders is reversed for analysis. Instead of calculating a required plastic moment for selection of girder sections, the plastic moment capacity is used to find maximum end moments for a beam to form a combined load mechanism. From formulae[8, 9] the maximum end moments for a selected beam can be found. These moments are called "limiting moments" and they are unique for each beam in that only these moments define a girder mechanism. Later in this paper, these are used to find the moment-rotation behavior of the beams of a sway subassemblage.

1.2 Purpose and Scope

It is the purpose of this thesis to improve the subassemblage method of analysis, the approximate analytical method for predicting the complete load-deflection behavior of a story of an unbraced frame, originally presented by Daniels[3, 4, 5, 6]. The modification to the method emphasizes that accuracy is improved and that the method is made more general, but that improvement will be within the scope of specific approximations. The extended method is therefore not exact but is shown to reasonably estimate strength and stiffness.

The original method is altered in two ways. First, the assumption for the points of contraflexure of columns is not restricted to midheight. This assumption is made general in order to handle most regions of a frame. Second, the original approach made use of charts to work the method by hand. Hand calculation is shown to be accomplished directly by tabular computation without using design charts. Also, Armacost[2] applied the subassemblage method to digital computor by using a small-step incremental approach. The direct computation worked for hand calculation is applicable to digital computer without need of the incremental approach.

The method is expanded to include the effect of axial shortening in the columns. Total frame behavior influences the strength and stiffness of each story through axial shortening. The inclusion of this effect for unbraced frames is important and makes the method more reliable.

Emphasis is made on understanding the method and applying it to engineering practice. Hence, this thesis solely intends to present refinements to the subassemblage approach to improve its accuracy and to give a reasonably easy method to apply.

2. THE SUBASSEMBLAGE METHOD OF ANALYSIS

This chapter will make modification to the subassemblage method of analysing frames as originally presented by Daniels. It will explain the method and the assumptions used. A generalization of the method enables it to be applied to most regions of a frame. The method will be described from the point of view that simplifying assumptions make direct tabular computation possible. Application to computer can allow the method to be used in a more sophisticated manner without as many simplifications.

2.1 The Assemblage

An assemblage is a model to represent the relationship between horizontal shear resistance and sway deflection for a particular story of a multistory frame. The assemblage consists of floor beams and a portion of columns below the floor level extending down to assumed inflection points as shown in Fig. 2.1. Different boundary conditions could be appropriate for modeling top, middle, or bottom stories. Past analyses of multistory frames[10, 11] have shown that most middle and lower stories had inflection points near midheight and actually most inflection points were above midheight. The method of analysis in this thesis uses the following assumption. Each column will have an inflection point at a distance αh below the centerline of the floor girders. The value of α is assumed to equal 0.5 for typical lower and middle stories.

2.2 Sway Subassemblages

The assemblage is further separated into smaller models called subassemblages. Each subassemblage consists of one column and either one or two floor beams framing into the column top as shown in Fig. 2.2. The far ends of the floor beams are assumed to sit on roller supports. Moment resistance of the far ends of the beams is simulated by springs, idealizing the beam to column restraint, and allowing beam end rotation. The sway subassemblage is the simplest possible model for calculating the load deflection behavior of a beam and column. Furthermore, the load-deflection curves of each subassemblage are combined to get the complete load-deflection behavior of the assemblage.

The influence of floor beams on the subassemblage is to provide stiffness which restrains the rotation of the column. The restraining moment on each beam is found using the following expression

$$M_B = K \frac{EI\theta}{L} \qquad 2.1$$

Where K = the stiffness factor for the beam. In this thesis the following assumptions are made concerning floor beams:

1. An elastic perfectly plastic moment-rotation relation is assumed.

2. The girder stiffness K is assumed to equal 6.0 to work the method by hand calculation. The stiffness is modified according to formulae by Danials[4] to work the problem by digital computer. If a plastic hinge forms at one end of the girder, the stiffness is reduced to 3.0 at the other end.

3. Girder rotation θ (clockwise rotation is taken as positive on the end of the member) is assumed equal for each end of the girders of a subassemblage. At formation of a plastic hinge, the rotation is assumed to be unrestrained for any further increment of rotation.

4. Beam axial shortening is neglected.

2.3 The Restrained Column

The column of a subassemblage is restrained by the floor beams and is permitted to displace laterally. Each column shear Q is expressed in an equilibrium equation for the freebody of the column shown in Fig. 2.3.

$$Q = \frac{M_u}{\alpha h} - P\left(\frac{\Delta}{h}\right) \qquad 2.2$$

where M_u = upper end moment on the column

P = column thrust

Δ = story lateral displacement

h = story height

α = decimal portion of story height from the top of the column to an assumed inflection point

Column end moments are determined from the equilibrium of moments at a joint shown in Fig. 3.4. The sum of column end moments qbove and below the joint equals the sum of beam end moments at the joint called restraining moment M_r.

$$M_r = M_{BL} + M_{BR} \qquad 2.3$$

where M_{BL} = girder end moment left of joint

M_{BR} = girder end moment right of joint

The column end moment M_u is assumed to be a portion β of the restraining moment.

$$M_u = \beta M_r \tag{2.4}$$

The value of β is assumed to equal - 0.5 for typical middle and lower stories. Daniels showed this assumption to be conservative. For analysing other than middle and lower stories, another assumption for β could be made.

The angles for the free body diagram in Fig. 2.3 of the restrained column have the compatibility relationship:

$$\frac{\Delta}{h} = \theta - \gamma \tag{2.5}$$

where θ = rotation of the joint

γ = the angle between the chord of the column segment and the tangent to the column centerline at the joint.

The rotation of the restrained column γ is a function of the column moment M_u and the axial thrust P. A method to relate the moment-rotation-thrust of the column required use of charts by Daniels. Armacost approximated this relationship by fitting an equation to the curves prepared by Daniels by assuming the initial slope of the curves closely approximated the ascending portion of the curves. These expressions have been considerably improved and have been made appropriate for strong and weak axis bending of wide flange sections by the following:

$$\gamma = J \times \frac{M_u}{M_{pc}} \tag{2.6}$$

where $$J = \left[\frac{\alpha h}{r_x}\left(25-22\frac{P}{P_y}\right) - 72\right] \times 10^{-5}$$

for strong axis bending

or $$J = \left\{\frac{\alpha h}{r_y}\left[19-16.5\frac{P}{P_y} + \left(\frac{P}{P_y}\right)^4\right] + 0.03\left(\frac{h}{r_y}\right)^2 - 6.0\right\} \times 10^{-5}$$

for weak axis bending

The thrust in each column is assumed to be constant for combined load analysis, as was done by Daniels. A preliminary design approach is used to find these column thrusts. A conservative estimate is made by assuming the girder end moment sum caused by lateral force to be distributed to each girder in proportion to their limiting end moment sum. The vertical end shears in the girders are thus found for lateral load by dividing the girder end moment sum by its length. These vertical shears are summed from the top floor downward and are added to the column thrusts based on grávity tributary area at each floor level to get the column thrusts for combined loading.

2.4 Solution Procedure for Subassemblage Analysis

The procedure to find the load-deflection behavior of an assemblage is next described. It is remembered that this method is a non-proportional displacement method because distributed gravity load is applied first and subsequent lateral load is found for a certain displacement. This method is not incremental as was presented by

Armacost. The procedure is summarized below in four steps:

1. Determine the initial moments on the beams under factored distributed gravity load. A one-story moment distribution is used to find the end moments under no lateral load. The limiting moments for the floor beams are found as described in Chapter 1. Each column limiting moment equals the reduced plastic moment under its factored axial force. The initial values of shear resistance Q are found for each column from Eq. 2.2 under no sway deflection.

2. Each subassemblage is analysed separately to find its load-deflection behavior. This step consists of determining the sequence of plastic hinge formation as each increment of rotation changes beam end moments from the initial state under vertical load to the final combined load state. The initial beam end moments are subtracted from the corresponding "limiting moments" to find the possible change in moment to form a plastic hinge. The amount of relative rotation θ necessary to cause this change is found by using Eq. 2.1. The minimum rotation for all beam ends controls and the controlling rotation is used to find the change of moments in the beams. These changes of beam moments are then added to the previous state to determine the new moments on the subassemblage. The restrained column end moment is found using Eq. 2.4. After formation of a plastic hinge at some point in the subassemblage, the stiffness is reduced to zero

at that point. This is the same as inserting a real hinge. A change of stiffness at other locations of the subassemblage may be necessary after a plastic hinge has formed. For example, if a plastic hinge forms at one end of a beam, the stiffness at the other end is reduced to K=3.0. The process of finding new moments and rotations for each successive plastic hinge is continued up to formation of a subassemblage mechanism. The rotations and column moments are saved and used in the next step.

3. The shear resistance and drift index are next calculated at formation of each plastic hinge. The drfft index is found using Eq. 2.5 where θ is known from (2) and γ is found using Eq. 2.6. Finally, the subassemblage shear resistance is calculated from Eq. 2.2, using the values of column moment from (2) and Δ/h from (3). The monotonic relationship of horizontal shear versus drift is available and shows the complete load-deflection response of the subassemblage.

4. The monotonic load versus drift curves are combined for all subassemblages to obtain the load-deflection curve of the assemblage.

It is noticed from the equation for equilibrium of the restrained column, Eq. 2.2, that the value of subassemblage shear resistance is conservative for an assumed point of contraflexure lower than the true point. From Eq. 2.2 the critical parameter is the value of α. For α greater than the true α, a calculated value of shear will be less than the true shear resistance. Thus, the calculated value for strength would be on the safe side. The choice of this parameter becomes very important in determining whether the analysis is higher or lower than the true solution.

To consider fixed base bottom story columns, the parameter α is also important. Under no lateral load, the point of contraflexure of a bottom story column may exist below midheight. Application of lateral load results in the point of contraflexure climbing towards the column top. Finally, the column may be bent in single curvature. For single curvature, an imaginary point of inflection actually lies above the column top. The combined load analysis presented in this thesis for a bottom story would be poor for a large variation of inflection point. But, a reasonable value of assumed α can give a safe analysis as long as it is assured that the true α is less than the assumed α. Then, even the case of single curvature bending would result in a conservative analysis.

For most middle and lower stories the assumption that $\alpha=0.5$ is conservative. This would lead to a conservative horizontal shear resistance but it remains to be seen whether sway deflection estimates are on the safe side. This thesis intends to show that sway deflection is affected by the influence of column axial shortening and previous uses of this method were unconservative as a drift estimate.

3. CHORD DRIFT IN THE SUBASSEMBLAGE ANALYSIS

This chapter will extend the analysis of the assemblage to include the effect of column axial shortening. The assumptions to separate the assemblage from the frame made the analysis very simple. They clearly emphasize conservatism in calculating the strength of the assemblage. First, the assumption that an inflection point is below the true inflection point can be conservatively made, and second, the assumption of the end moment in the restrained column is conservative by taking a safe proportion of the restraining moment M_r. The use of these assumptions also makes a sway deflection estimate conservative for only the assemblage but if the action of the frame below the assemblage is considered, the resultant sway deflection will increase. Also, additional moments are caused by differential column axial strain. This chapter describes an approximate method to include the effect of chord drift in the subassemblage analysis.

3.1 Regions of the Frame Affected

The major influence of chord drift would be on relatively tall frames and mainly in the middle and upper regions of multistory frames. As pointed out by Kim[10], the influence of chord drift was minimal under nonproportional loading for low frames but was significant in a 26 story frame. Also, Parikh[11] demonstrated its effect to be significant for a 24 story frame. The height to width ratio for which chord drift would have importance would be difficult to find, therefore, each frame should be checked individually to assign relative importance to this effect.

It is proposed that chord drift should be included in the analysis of frames of relatively high vertical dimension. It is described herein how to include the effects of chord drift in the sway subassemblage method of analysis.

3.2 Effect of Axial Shortening of Columns

The action of column axial shortening affects the strength and stiffness of an assemblage in two ways. First, the differential shortening of columns causes drift in an assemblage due to a geometric change of the frame below the story being considered. As shown in Fig. 3.1, the exaggerated shortening of column ED has caused the top of the frame BCFE to deflect horizontally relative to story ABED. This horizontal deflection is caused by a geometric change in the frame. With a much larger and complex frame, differential shortening will not be as easily scrutinized as in this two story example but will vary from column to column.

To account for sway deflection in an assemblage caused by this geometric effect, a virtual work method is used. It is assumed that the longitudinal strain in the columns is elastic under all loading cases. The foundation of the virtual work method lies in establishing an equilibrium system for the structure under unit loading. A deformation system, resulting from actual loading, is superimposed onto the equilibrium system to find the defection of the frame.

The actual axial load in the columns is constantly varying under proportional application of lateral load. For simplicity, it is assumed that the axial force in the columns is constant under combined gravity plus lateral load. The column thrust for combined load is found, using the concept in Chapter 2, by distributing the moments in a story caused by lateral load in proportion to their limiting end moments. The resulting column thrusts P are achieved only for combined load mechanism, and they are considered to be a conservative set of forces. Then, the actual column strain equals

$$\epsilon = \frac{P}{AE} \tag{3.1}$$

The equilibrium system is formulated by applying a unit lateral load to the floor level under investigation and a unit lateral load is applied to the next lower floor level in the opposite direction, shown in Fig. 3.2. By applying the unit loads in this manner, a deflection of only one story is found. This deflection is a relative deflection of one floor level to the next lower floor level. The column axial forces due to the unit load system are found using an approximate method developed by Spurr[12]. The axial force in a column is taken as proportional to the relative column areas and the distance from the column group centroidal axis. This axial force N of the equilibrium system in one line of columns is assumed equal for all columns. Thereby, the relative horizontal story deflection Δ is a summation of axial force due to unit load times the actual elastic strain of each column up to the floor level that deflection is required as expressed by the following:

$$\Delta = \sum_{j=1}^{\text{no. of col.}} N_j \sum \left(\frac{Ph}{AE}\right)$$

This relative deflection due to column axial shortening has been determined for the factored combined loading case. To use this deflection at working load, it is assumed that the deflection due to axial shortening is divided by the load factor (F=1.3). Furthermore, after reaching the factored lateral load, the deflection due to axial shortening is assumed constant. This deflection can be used in the column equilibrium equation Eq. 2.2 under the above assumptions. Relative magnitudes of Eq. 2.2 show that shear resistance is not significantly reduced when this effect is included.

The second part for the inclusion of axial shortening in the subassemblage method of analysis is described herein. As shown in Fig. 3.3, if CD settles a distance Δ_v, the frame ABCD is subjected to sway and the members are subjected to bending beyond any loading condition. This is similar to column shortening in a multistory frame. Application of lateral load causes differential joint displacement, resulting from column shortening, and this joint displacement causes additional moment.

Under combined loading, the previously mentioned assumption of constant axial force results in constant joint settlement due to column axial shortening. The amount of differential settlement Δ_j of joints gives fixed end moments from the equation

$$FEM = \frac{6EI}{L^2} \Delta_j$$

For each assemblage, a one-story moment distribution will give a final moment diagram for the moments caused by axial shortening. To include these moments in the subassemblage analysis, it is assumed that these moments are subtracted from the limiting moments to give new "limiting end moments". These new limiting moments would be used in the same procedure for executing the analysis as described in Chapter 2.

To demonstrate the use of the subassemblage analysis as described in Chapter 2 and to show the effects of column axial shortening, and illustrative example is provided in the following chapter.

4. NUMERICAL EXAMPLE

The sway subassemblage method described in the preceeding chapters is next used to analyse an example frame. Several load deflection curves of one-story assemblages will be presented for comparison with corresponding story behavior calculated by other methods.

The frame shown in Fig. 4.1 uses the geometry and loading of the design example given in Ref. 1. The original example showed the plastic design of a braced frame; the example in this thesis shows a new design made by plastic procedures[9] of an unbraced frame. It is a three-bay, twenty-four story frame with distributed loads using steel with a 36 ksi stress level, and it is designed such that the bare steel skeleton is required to carry all loadings. The beams were designed using clear span length and live load reduction was considered for both beams and columns. The design ultimate load is equal to 1.3 times the working load, corresponding to the combined loading condition, and is equal to 1.7 times the working load, corresponding to the gravity loading case. The design of this frame is preliminary, and as such, secondary checks have as yet not been made. The member sizes are shown in Tables 4.1 and 4.2 and the gravity loads are shown in Tab. 4.3. The design is used to check the suitability of the subassemblage method to analyse frames.

4.1 Comparisons To Be Made With Other Methods

To demonstrate effectively the use of the assemblage for evaluation of frame strength and drift, the subassemblage method is compared to the sway increment method[10] and Parikh's elastic-plastic analysis of unbraced frames[11]; both developed at Lehigh University. The sway increment method provides for the entire frame a nonproportional loading analysis which gives the lateral shear resistance of the frame and sway deflection of stories as plastic hinge formation progresses to the point of frame failure. Parikh's analysis provides a proportional loading analysis of the entire frame, and in this thesis it is used to check the working load deflections of individual stories.

The comparisons of these analyses will consider strength and story drift as the main criteria for adequately judging frame behavior. Working load deflection is the main concern for judging the story drift but this criterion is limited because the largest permissible drift index is uncertain. In this thesis any reasonable drift index will be considered to be acceptable. The criterion for strength for combined load analysis is the amount of shear resistance available in the frame. The shear resistance for an assemblage should be greater than the design ultimate shear for each floor level. To be conservative the shear resistance found by the subassemblage method should be less than that found by overall frame analysis.

4.2 Results of Column Axial Shortening

Inclusion of column axial shortening in the subassemblage analysis has been shown to be significant in earlier references. To demonstrate the effect of chord drift using the subassemblage method of analysis, floor level 14 of the example frame is analysed both neglecting and including this effect. In Fig. 4.2 the load-deflection curves are shown, in which the load Q, plotted on the vertical axis, is the shear resistance of the assemblage and the drift index, plotted on the horizontal axis, is the horizontal sway deflection of a story divided by the story height below the floor level. The two curves are very similar in shape. The inclusion of chord drift shows an increase in sway deflection between 20 and 30% which is substantial and therefore should not be neglected. The maximum shear resistances for the two curves are nearly the same. The strength of the assemblage decreases only slightly when chord drift is included and thus overall strength is not significantly affected. The successive formation of plastic hinges for each subassemblage is shown in Fig. 4.3. No change in plastic hinge formation occurs in subassemblages A or D. In subassemblages B and C, it is noticed that inclusion of chord drift, caused the exterior beams of the two subassemblages rather than the interior beam to form plastic hinges first. This results from the decrease in positive beam end moment of the interior beam due to differential shortening of the interior columns; thus it took much more positive rotation θ to form plastic hinges in the interior beam. With increased positive rotation the exterior beams have plastic hinges form first. Hence, the effect of column differential shortening on an assemblage is an increase in horizontal sway deflection and overstressing occurs at different locations. Similar results were obtained for other floor levels.

4.3 Results of Subassemblage Analysis Compared to Other Methods

Next, the load-deflection curves for several different floor levels are compared using the simplified subassemblage approach and using the two frame analyses described earlier. For convenience, the story behaviors for these analyses are compared in three parts of the frame: the lower, middle and upper stories.

4.3.1 Middle and Lower Stories

In Fig. 4.4 the load-deflection curve of floor level 19 compares the subassemblage method to the sway increment method of overall frame behavior and Parikh's working load deflection. All analyses include the effect of column differential shortening. For the subassemblage method, the values of α and β of Eqs. 2.2 and 2.4 are assumed to equal 0.5. Excellent agreement is evident. The sway increment method shows larger shear resistance up to the point where the frame lost stiffness and the curve is discontinued, telling nothing further about the story behavior. The subassemblage analysis yields a lower shear strength as expected from the conservative assumptions. The subassemblage approach also provides the complete curve through unloading of the lateral force, and thus shows how the story behaves when separated from the frame. The working load deflection is the same for both methods and is slightly less than the drift index given by Parikh's analysis.

The load-deflection curves for levels 17 and 14 are shown in Fig. 4.5 and 4.6, respectively. Both curves of the subassemblage approach show good agreement with the sway increment method. The maximum lateral load is less for the subassemblage approach which again shows expected conservatism in the assumptions. The subassemblage method still indicates strength greater than the design ultimate load, showing that the story legitimately satisfies the strength criterion. The working load sway deflection of level 17 for the subassemblage approach is the same as for Parikh's analysis although less than for the sway increment method. For level 14 the working load sway deflection is slightly greater for the subassemblage method than for Parikh's analysis. These results show that story deflection is very closely approximated by the subassemblage approach.

The load-deflection curves for floor level 10 are shown in Fig. 4.7. The value of β is assumed to equal 0.6 for this level. The subassemblage analysis is conservative in showing less strength than the sway increment analysis and also is conservative in showing a greater working load deflection than either Parikh's analysis or the sway increment method. The slight disagreement of results at this level is partly due to the conservative assumption of inflection point at midheight. Also, the inclusion of increased sway caused by chord drift is conservative.

4.3.2 Upper Stories

The design of upper stories is generally controlled by the gravity loading case and, as such, combined load analysis may not be necessary. However, the transition from the gravity load controlled region to the combined load controlled region may not be readily apparent. To better analyse the upper stories for the combined loading case, different assumptions for α and β may be necessary.

The load-deflection curve for floor level 6 is shown in Fig. 4.8. For the subassemblage approach, the value of α was assumed to equal 0.75 and the value of β was assumed to equal -0.5. The subassemblage analysis shows a slightly larger shear resistance and it is in very good agreement with the sway increment method. The working load deflection is the same as Parikh's analysis. The assumption of α proves to be acceptable in giving a good subassemblage analysis for this region of the frame.

The load-deflection curve for floor level 2 is shown in Fig. 4.9. The value of α is assumed to equal 0.75 which is the same as for level 6. The initial part of the curve agrees exactly with the sway increment analysis and the working load drift index is the same as Parikh's analysis. The maximum lateral load of the subassemblage approach is much larger than from calculations in the sway increment method. The sway increment method stopped earlier due to frame failure at another location in the frame, resulting in a lack of information of the complete behavior at this level. Thus, the subassemblage method gives a complete curve, although it is uncertain if the true behavior is given beyond the results of the sway increment method. However, so near the top of the frame, the combined load analysis need not be exact because this region is undoubtably controlled by gravity loading.

A comparison of column axial thrusts is made at factored combined gravity plus lateral load in Tab. 4.3 for the several analyses of floor levels 2, 5, 10, 14, 17, 19. The column thrusts were computed by Parikh's analysis and by the estimate for subassemblage analysis given in Chapter 2. By the subassemblage analysis these thrusts were considered constant. The thrusts from both methods are reasonably close. The thrusts in the leeward interior and exterior columns are greater for the subassemblage analysis. The total sum of column thrusts being constant for each floor level results in the windward interior and exterior columns carrying less thrust for the subassemblage method. The leeward columns therefore will show conservatism when axial thrust is considered in the subassemblage approach.

A comparison of the cumulative shortening of the columns below a certain level is made in Tab. 4.4 for factored combined gravity plus lateral load. The two methods compared are the Parikh analysis and the estimate for the subassemblage approach. The downward joint displacements are very close to one another because the axial forces in columns are close by both methods. This indicates that the differential joint displacements will be fairly accurate in finding fixed ended moments due to column axial shortening.

4.4 One Story Assemblage Method of Analysis

Along with the sway increment analysis, a one story assemblage method was developed by Kim[10]. This approach has a similar assumption to the subassemblage approach; it assumes the location of inflection point at midheight of the columns. This is more restrictive than the subassemblage approach presented in this paper although it has been shown to be acceptable for middle and lower regions of the frame.[4] A comparison of the one-story assemblage analysis to the subassemblage analysis is shown in Fig. 4.10. where the load-deflection curves of floor level 14 are plotted. Previously, the maximum lateral load for the subassemblage analysis was generally less than for the entire frame analysis. The one-story assemblage analysis shows a greater reduction in predicted shear strength and is therefore more conservative than the subassemblage analysis. Analyses of other middle and lower levels also indicate greater conservatism in this approach.

4.5 Summary

In this comparative study, it is noticed that the subassemblage analysis showed very good agreement with more exact full frame analyses. This analysis is crude due to simplifying assumptions and, as such, it should not be expected to be exact. The results of the comparisons show that the extended but simplified method shows conservatism for both strength and sway deflection. It, therefore, proves that if a floor level analysed by this approach is found to be satisfactory, then the level will be acceptable in context with the entire frame.

5. SUMMARY AND CONCLUSIONS

It has been the purpose of this thesis to extend the subassemblage method of analysing unbraced frames. The analysis is used to predict elastic-plastic second order behavior under non proportional loading. A general procedure is made by including several effects not included in the original development of the method. The possible use of different than midheight points of inflection for separating an assemblage from the frame was developed. This could allow use of the subassemblage method for upper stories or bottom story as well as for middle and lower regions which use the midheight as the inflection point. The effect of column axial shortening was extended to the subassemblage analysis. The sway subassemblage method is considerably simplified with the use of a direct computation for finding the minimum rotation at formation of each successive plastic hinge. This is applicable to hand computation or digital computer usage replacing the highly inefficient incrementation of rotation by very small amounts as suggested in Ref. 2 to find the complete load-deflection curve of an assemblage.

From the analytical results of a design example given for an unbraced multistory frame, it can be concluded that:

1. The effect of column axial shortening is not considerable in changing the lateral load capacity of an assemblage. The major effect is an increase of lateral deflection, and over-stressing at different parts of the assemblage causes a change in the order of plastic hinge formation.

2. A choice of point of inflection at midheight makes the subassemblage approach conservative for middle and lower stories. For other than lower and middle stories, a choice of α below midheight helps to make the analysis more reliable. Guaranteed conservatism is possible for a choice of inflection point below the actual point of inflection.

3. A comparison with more exact overall frame analyses shows that the extended subassemblage approach is conservative in its estimation of both lateral load capacity and horizontal sway deflection. Therefore, the simplified approach can be used to evaluate the behavior of unbraced frames.

6. NOMENCLATURE

A	Area of wide flange sections;
E	Modulus of Elasticity;
FEM	Fixed end moment;
h	Story height;
J	Term relating moment-rotation-thrust of a column;
I	Moment of inertia;
L	Span length;
M_B	Beam end moment;
M_{pc}	Reduced plastic moment;
M_u	Column end moment;
P	Column thrust;
P_y	Column axial yield load;
Q	Horizontal shear force on a column;
r_x, r_y	Column radius of gyration, x-axis and y-axis;
Σ	Finite summation;
α	Decimal portion of story height from the column top to an assumed inflection point;
β	Decimal portion of the sum of beam end moments at a joint, assumed to equal the column top moment;
γ	Column chord rotation;
Δ	Horizontal displacement of the column top relative to the column bottom;
ϵ	Axial column strain;
θ	Rotation of the joint.

7. TABLES

Table 4.1 Beam Sections for Example Frame

Level	AB and CD	BC
1	W16x31	W10x21
2	W16x31	W12x22
3	W16x36	W12x22
4	W16x36	W14x22
5	W16x36	W14x26
6	W16x36	W14x26
7	W16x40	W14x26
8	W16x40	W14x26
9	W16x40	W14x26
10	W18x40	W16x26
11	W18x40	W16x26
12	W18x45	W18x35
13	W18x45	W18x35
14	W18x45	W18x35
15	W21x44	W18x40
16	W21x44	W18x40
17	W21x44	W18x40
18	W18x55	W18x55
19	W18x55	W18x55
20	W18x55	W18x55
21	W21x49	W21x49
22	W21x49	W21x49
23	W21x55	W21x55
24	W21x68	W21x68

Table 4.2 Column Sections for Example Frame

Level	A and D	B and C
1	W12x40	W12x40
2	W12x40	W12x40
3	W12x40	W12x40
4	W12x40	W12x40
5	W12x58	W12x58
6	W12x58	W12x58
7	W12x79	W12x79
8	W12x79	W12x79
9	W12x85	W12x85
10	W12x85	W12x85
11	W14x111	W14x111
12	W14x111	W14x111
13	W14x111	W14x119
14	W14x111	W14x119
15	W14x127	W14x136
16	W14x127	W14x136
17	W14x150	W14x158
18	W14x150	W14x158
19	W14x158	W14x176
20	W14x158	W14x176
21	W14x176	W14x193
22	W14x176	W14x193
23	W14x219	W14x237
24	W14x219	W14x237

Table 4.3 Working Gravity Loads For Example Frame

(a) Beam Loads (K/ft)

Level	Ab and CD	BC
1	2.34	2.34
2	2.32	3.94

(b) Joint Loads (kip)

Level	A and D	B and C
1	8.63	2.43
2	20.39	3.05
3-22	15.97	-1.19*
23-24	16.46	-0.89*

*Due to live load reduction in columns

Table 4.4 Axial Loads on Columns for Factored Combined Loading

Level	Column Axial Load Below Level[A]			
	Col. A (kips)	Col. B	Col. C	Col. D
2	97.4	114.4	102.8	103.1
6	282.2	324.9	308.2	319.5
10	459.6	519.7	540.2	556.0
14	626.3	684.1	804.9	803.2
17	745.9	786.7	1023.7	995.7
19	820.7	837.1	1188.3	1127.0

Level	Column Axial Load Below Level[B]			
	Col. A (kips)	Col. B	Col. C	Col. D
2	98.2	105.7	110.8	103.1
6	280.8	298.3	331.7	325.2
10	450.5	490.0	565.9	571.6
14	604.4	657.2	826.8	833.6
17	711.4	760.3	1043.7	1040.6
19	778.9	810.8	1207.2	1181.1

A Parikh's Analysis

B Estimate for Subassemblage Analysis

Table 4.5 Cumulative Shortening of Columns for Combined Loading

Level	Displacement Below Level[A]			
	Jt.1 (inches)	Jt.2	Jt.3	Jt.4
2	1.53	1.57	1.89	1.95
6	1.33	1.34	1.68	1.73
10	1.07	1.05	1.4	1.43
14	0.8	0.75	1.07	1.10
17	0.58	0.52	0.80	0.81
19	0.44	0.39	0.62	0.62

Level	Displacement Below Level[B]			
	Jt.1 (inches)	Jt.2	Jt.3	Jt.4
2	1.49	1.51	1.97	2.03
6	1.27	1.28	1.73	1.80
10	1.02	1.01	1.43	1.50
14	0.76	0.73	1.09	1.15
17	0.54	0.51	0.81	0.85
19	0.41	0.38	0.62	0.65

A Parikh's Analysis

B Estimate for Subassemblage Analysis

8. FIGURES

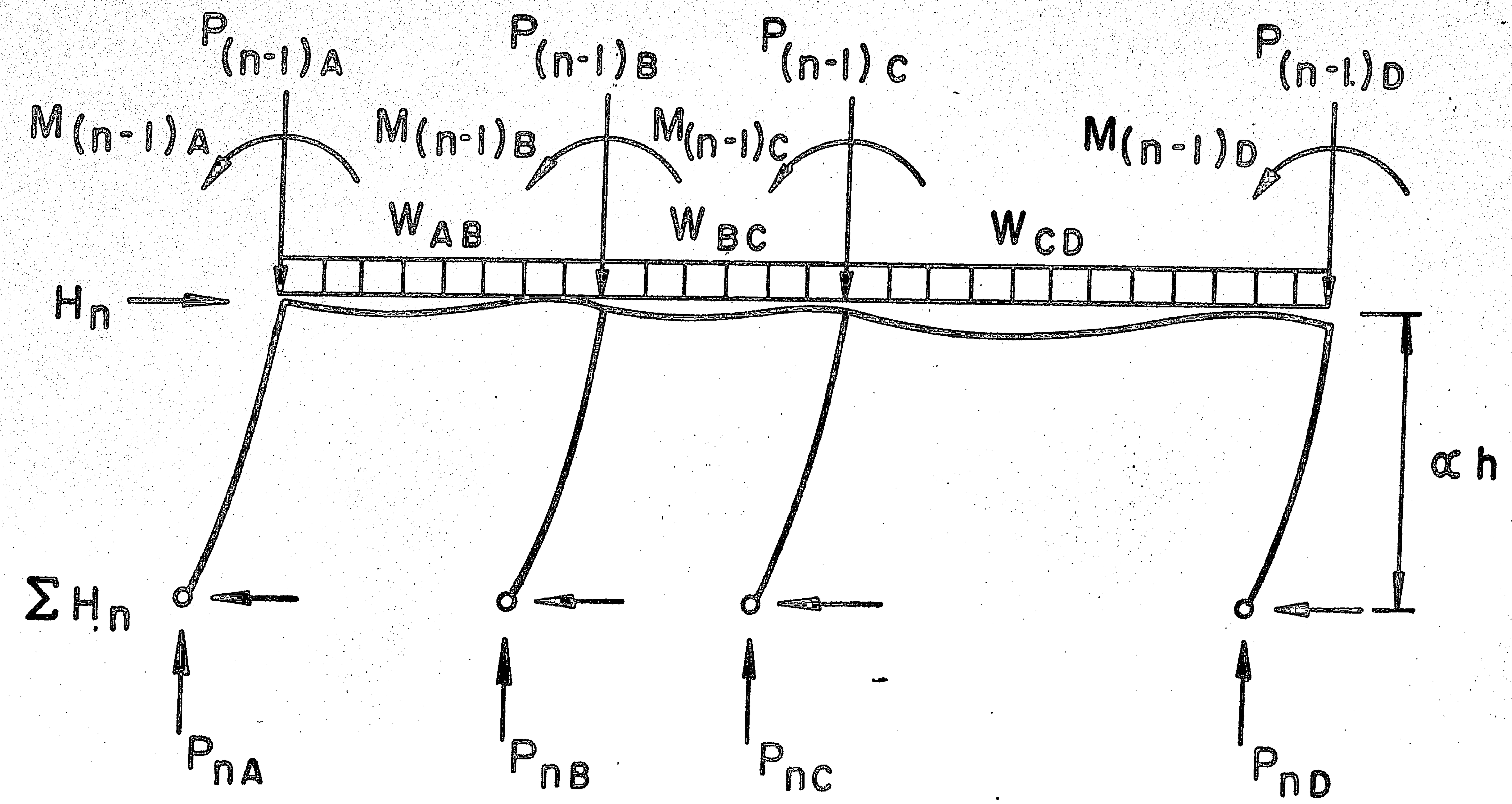

Fig. 2.1 One-Story Assemblage

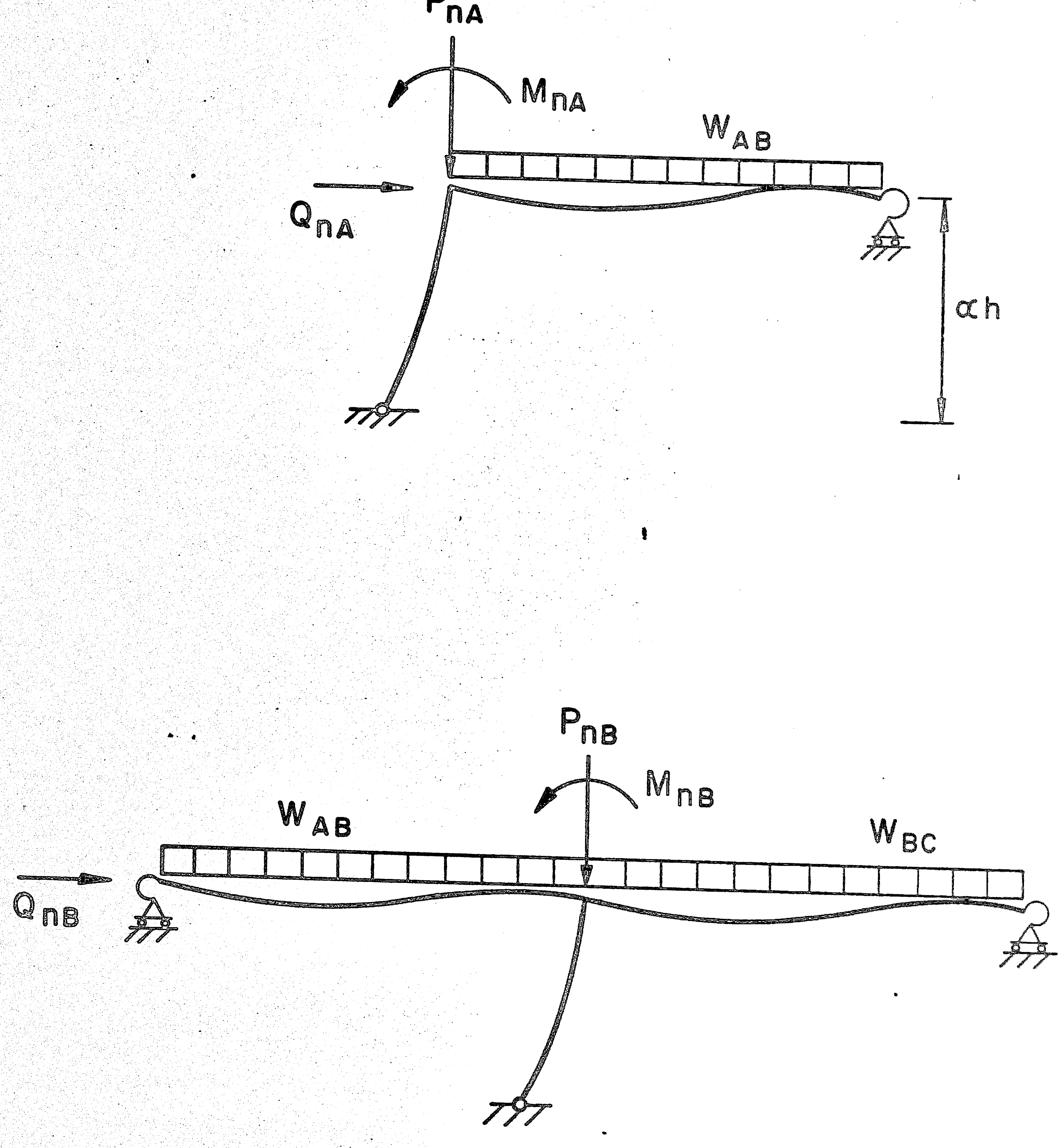

Fig. 2.2 Typical Subassemblages

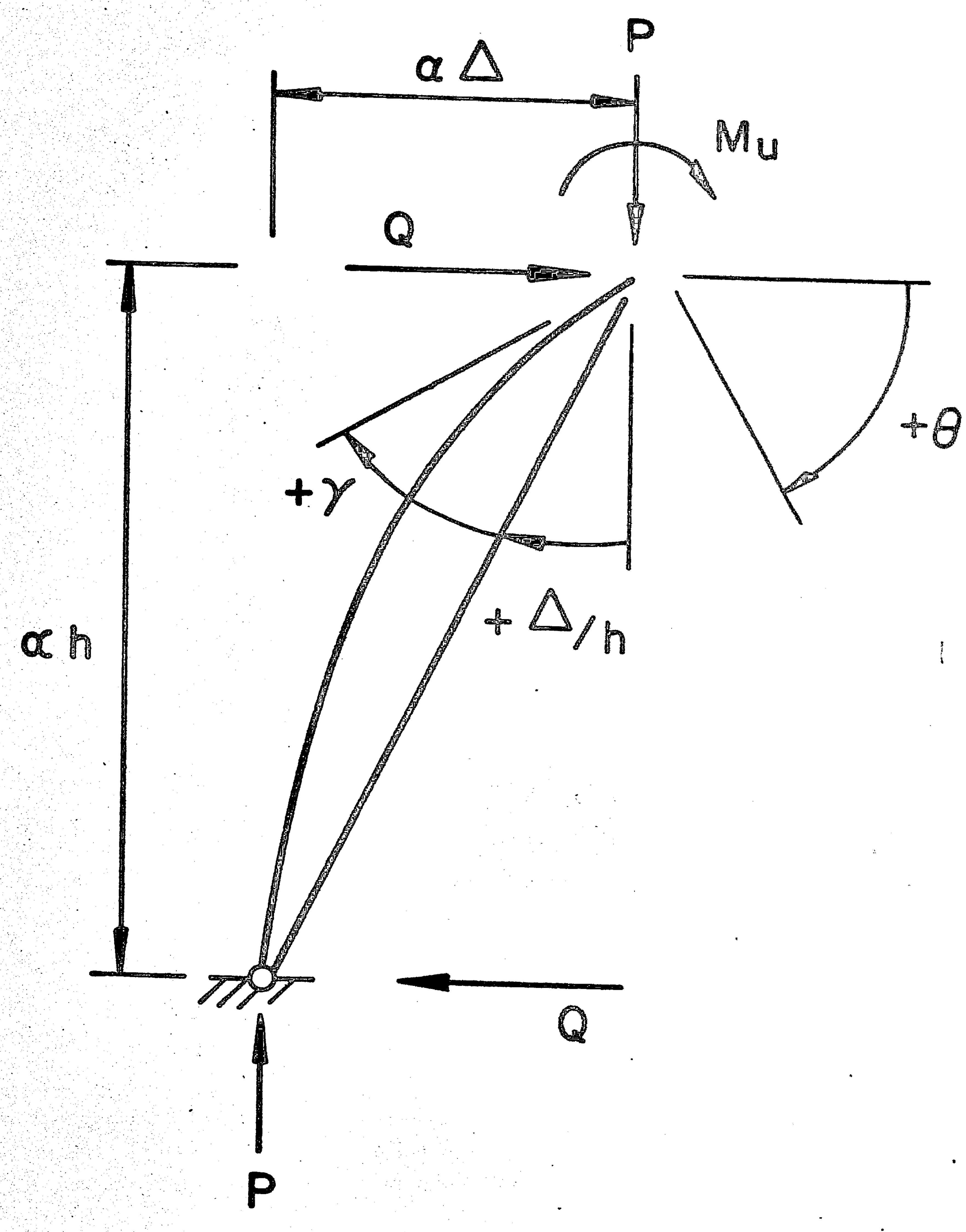

Fig. 2.3 Restrained Column

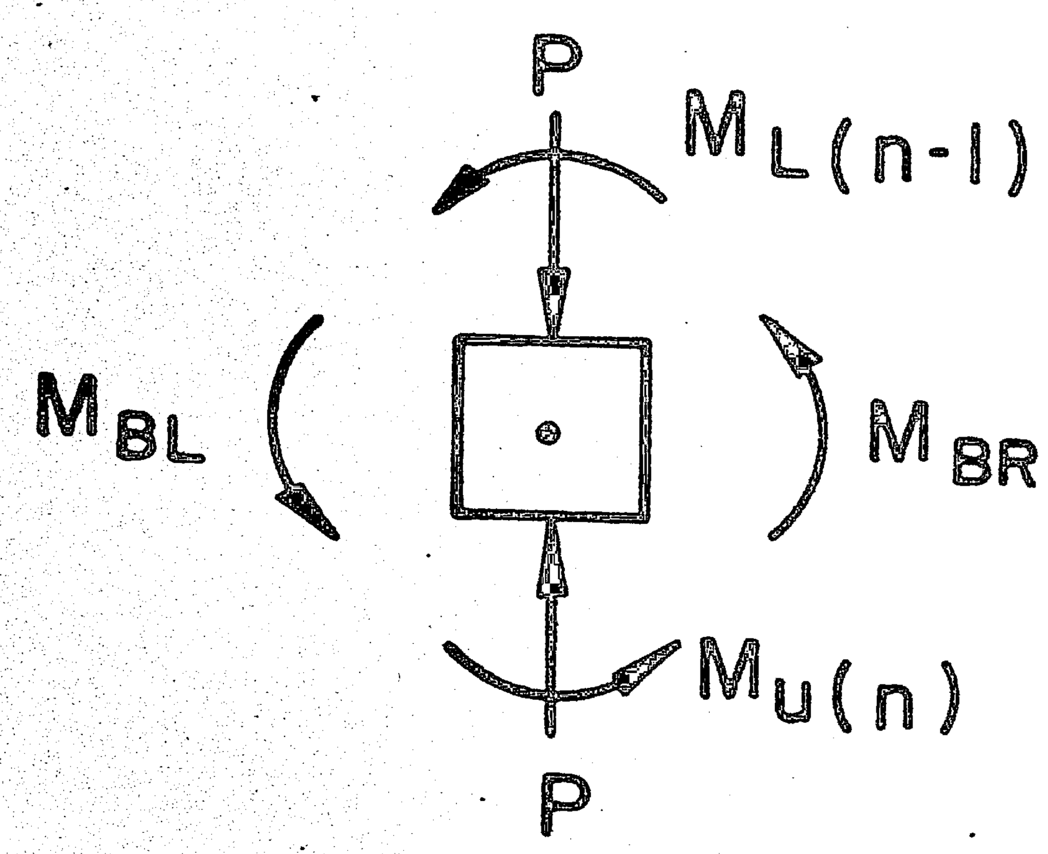

Fig. 2.4 Equilibrium of Moments at Joint

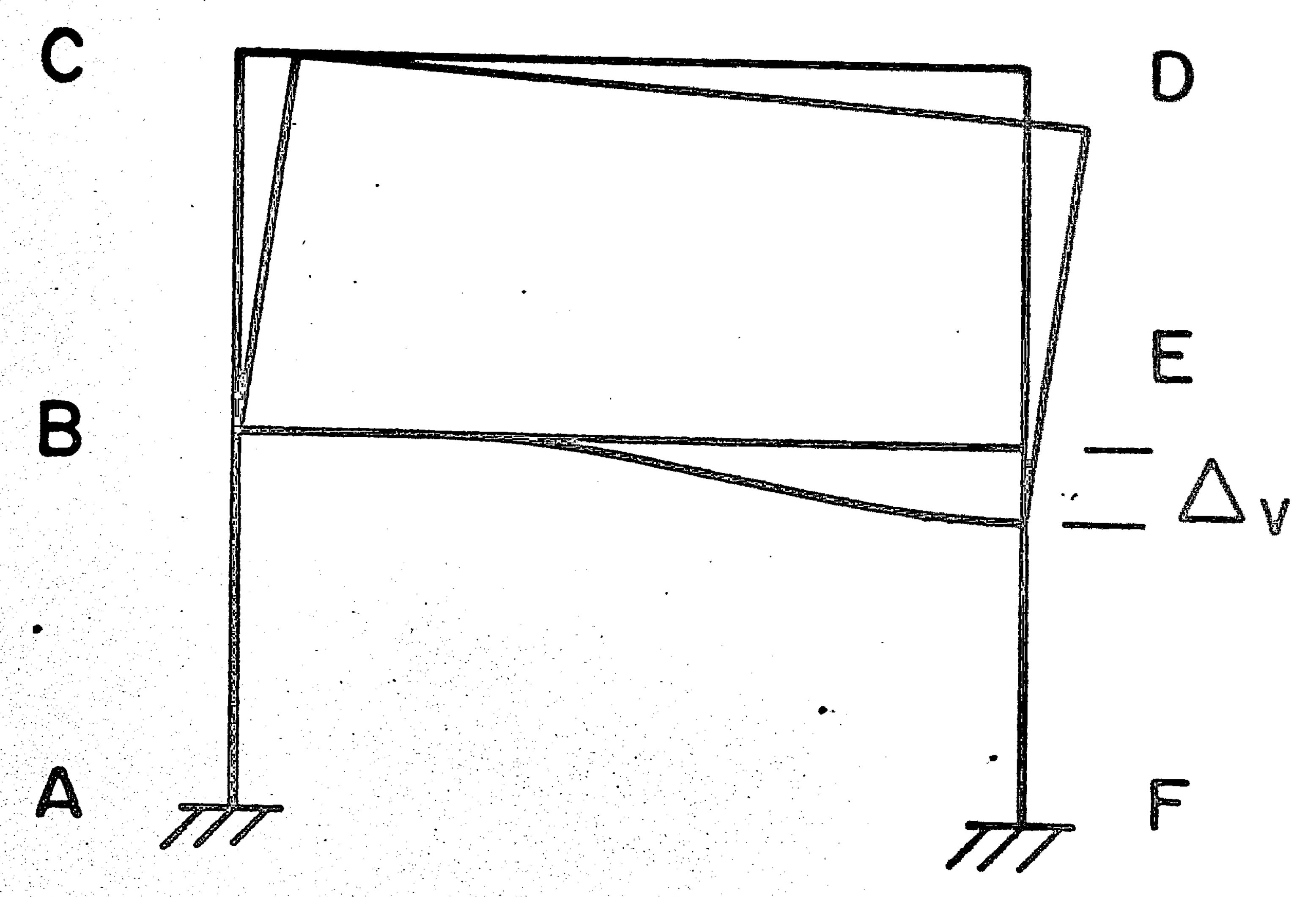

Fig. 3.1 Column Axial Shortening Causes a Frame Geometry Change

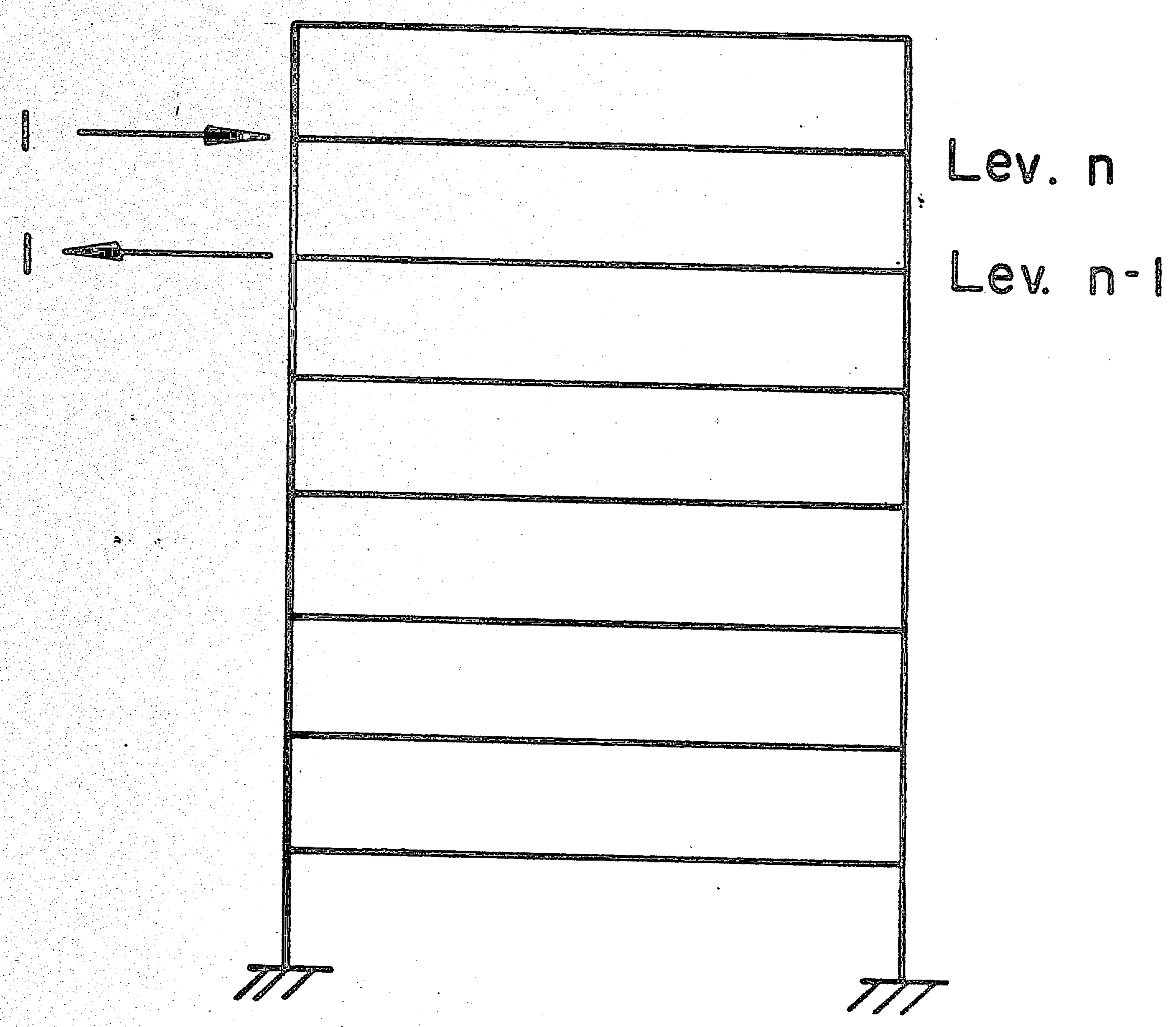

Fig. 3.2 Application of Dummy Unit Load System

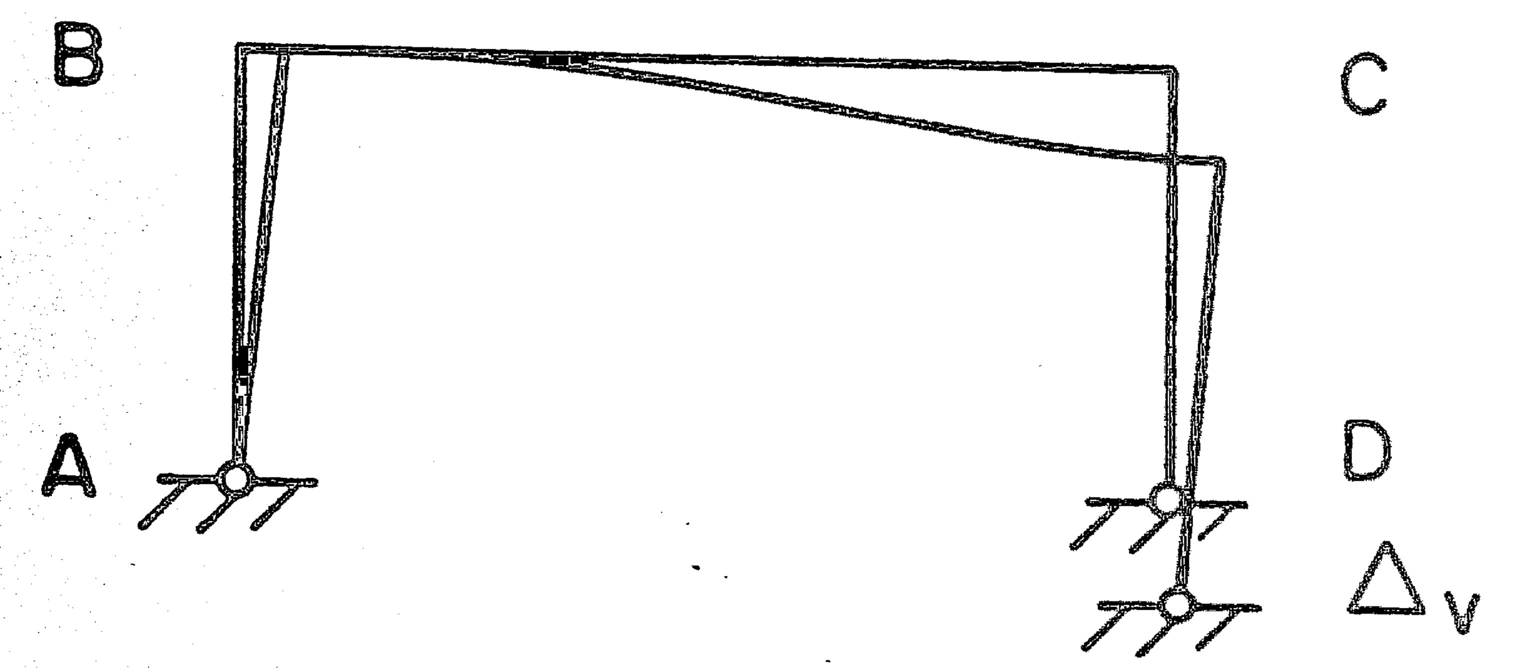

Fig. 3.3 Settlement at D Causes Frame Change

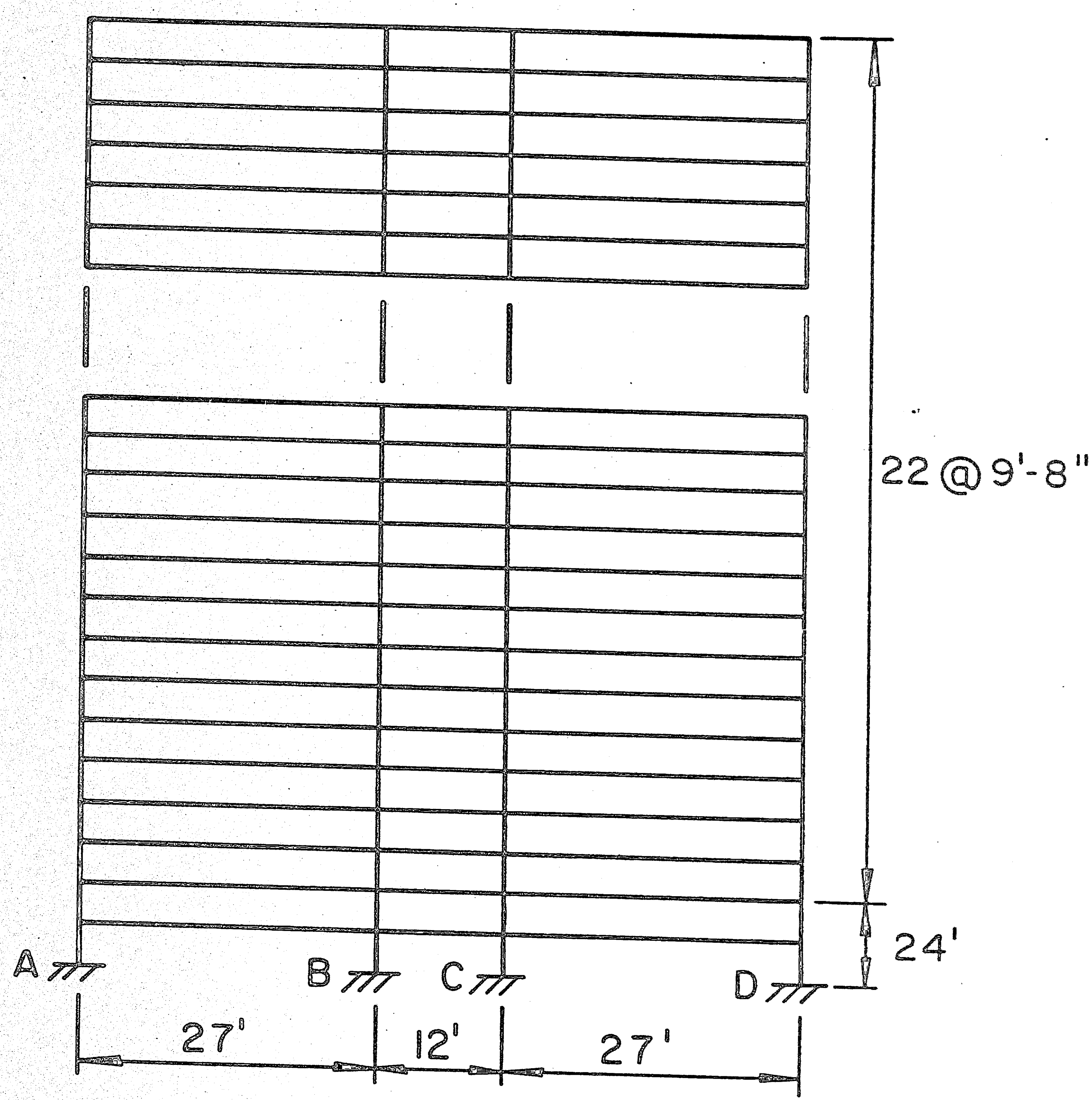

Fig. 4.1 Example Frame

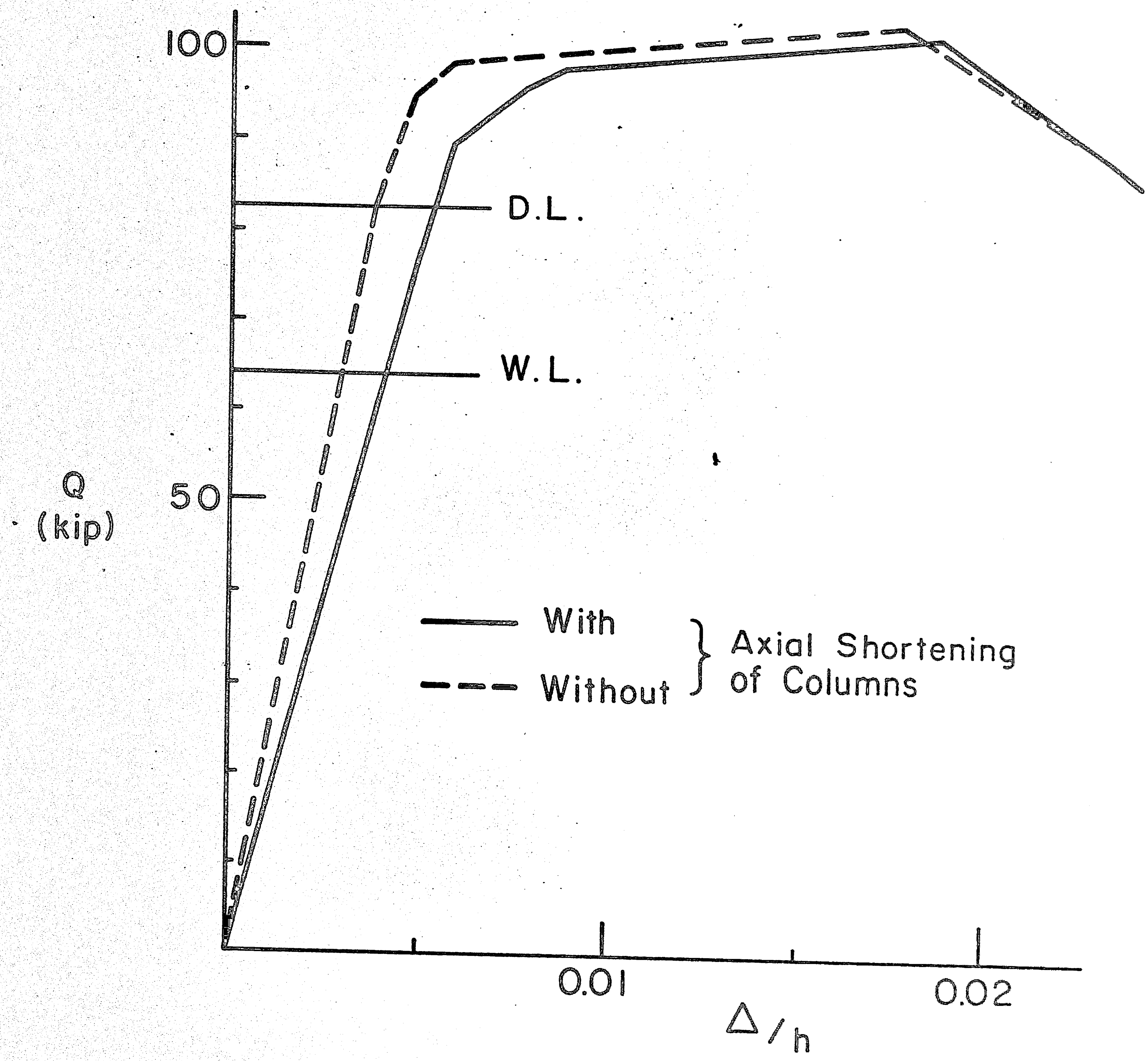

Fig. 4.2 Load-Deflection Curve of Level 14

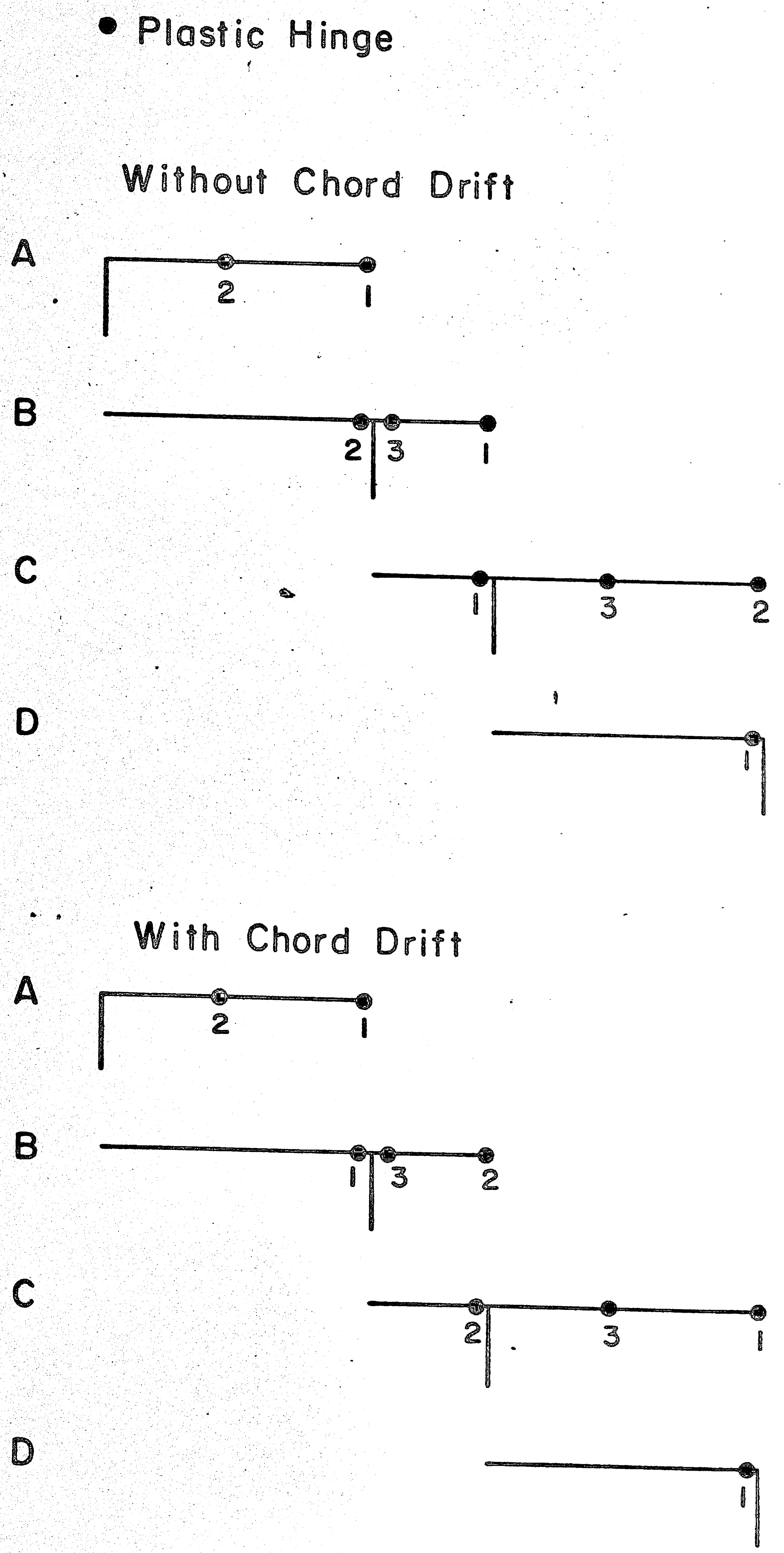

Fig. 4.3 Order of Plastic Hinge Formation for Level 14

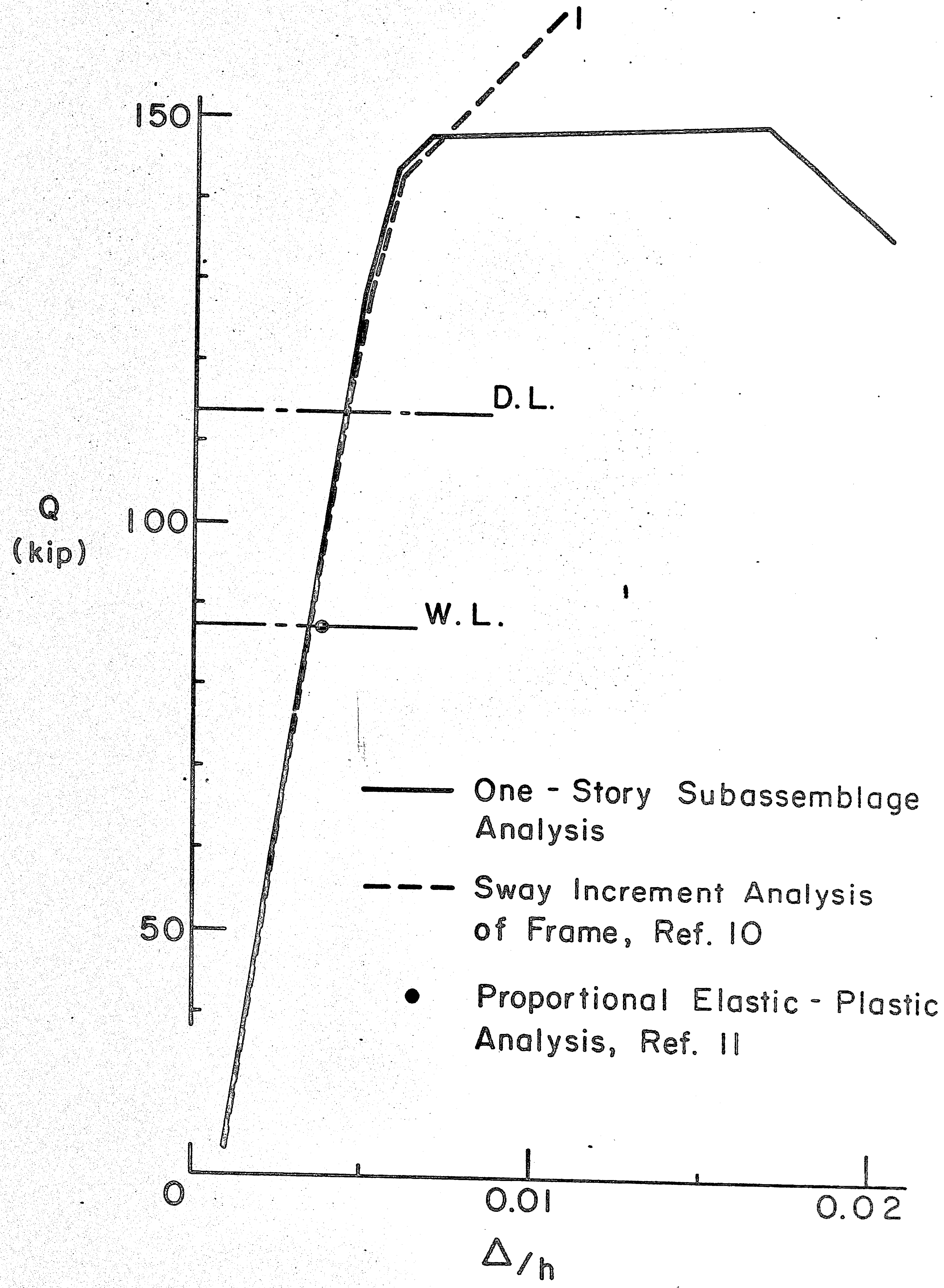

Fig. 4.4 Load-Deflection Curve of Level 19

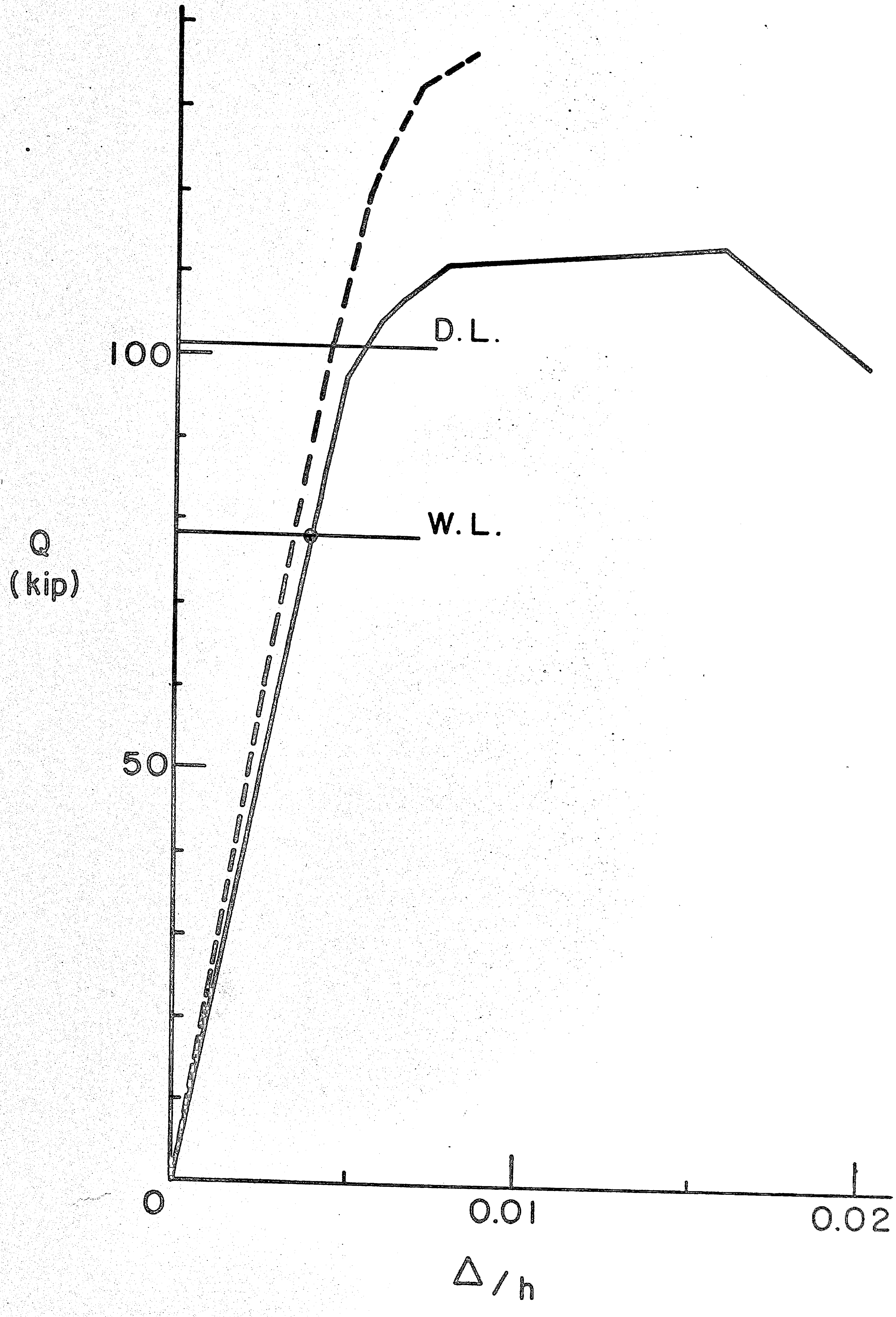

Fig. 4.5 Load-Deflection Curve of Level 17

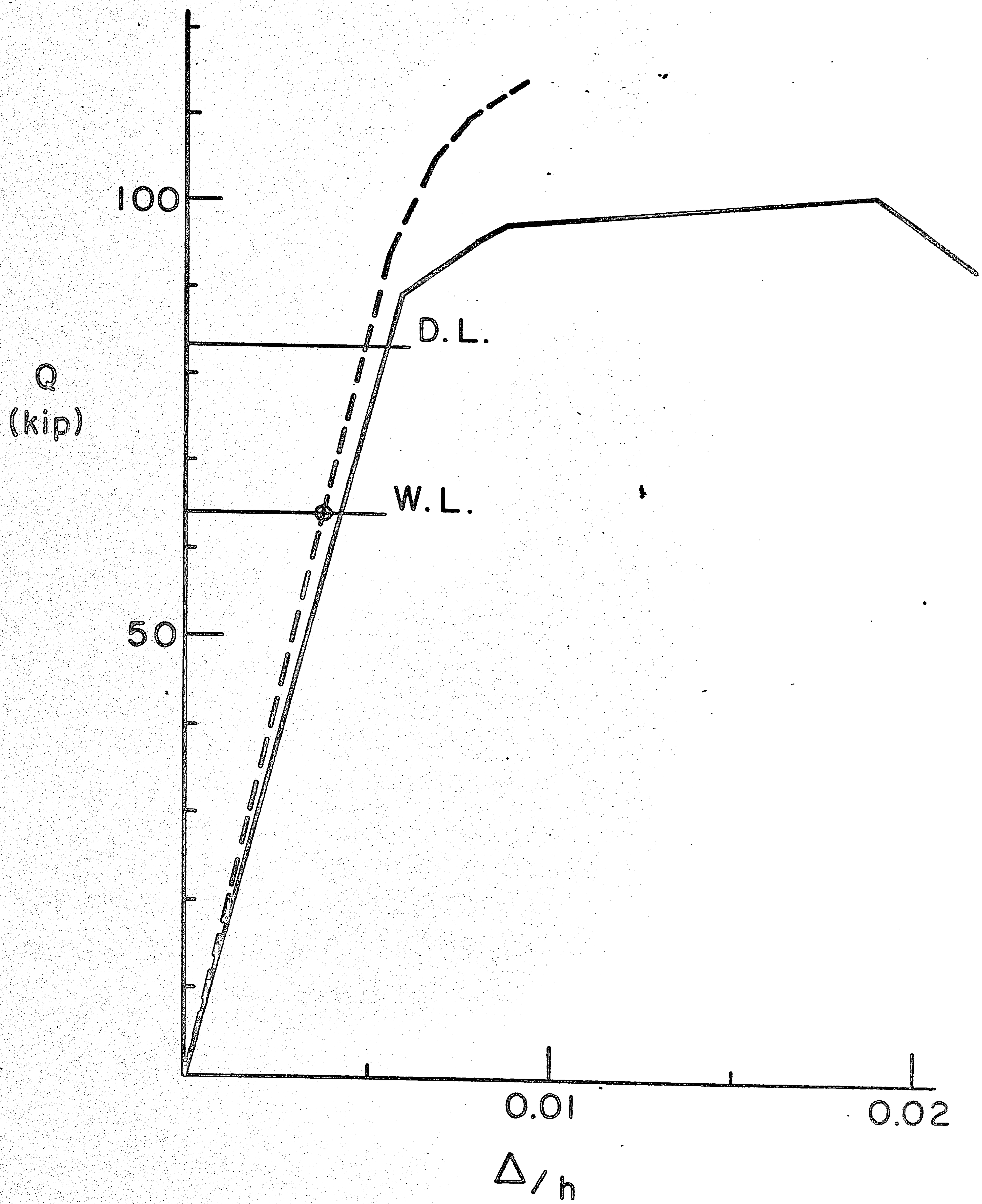

Fig. 4.6 Load-Deflection Curve of Level 14

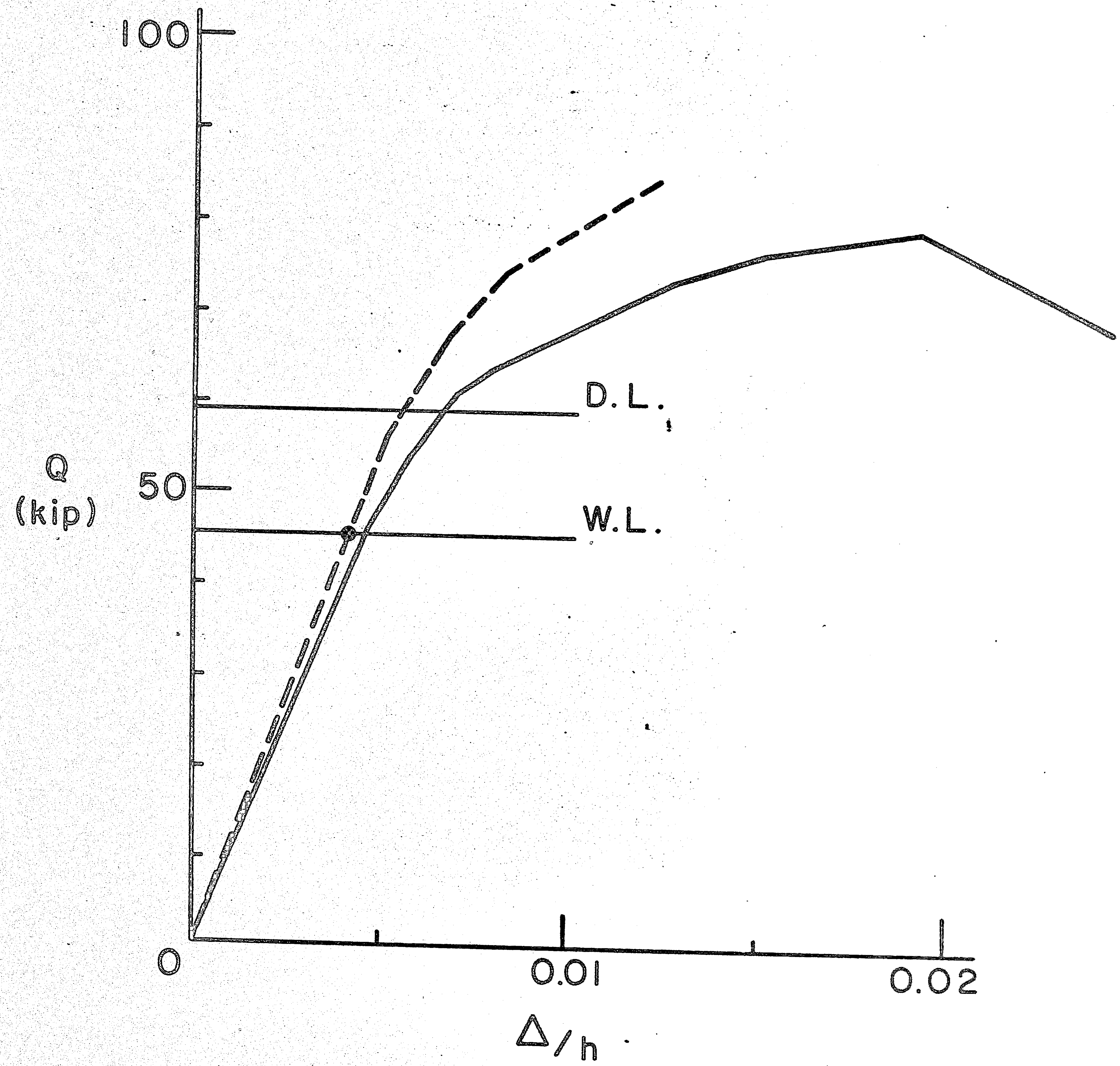

Fig. 4.7 Load-Deflection Curve of Level 10

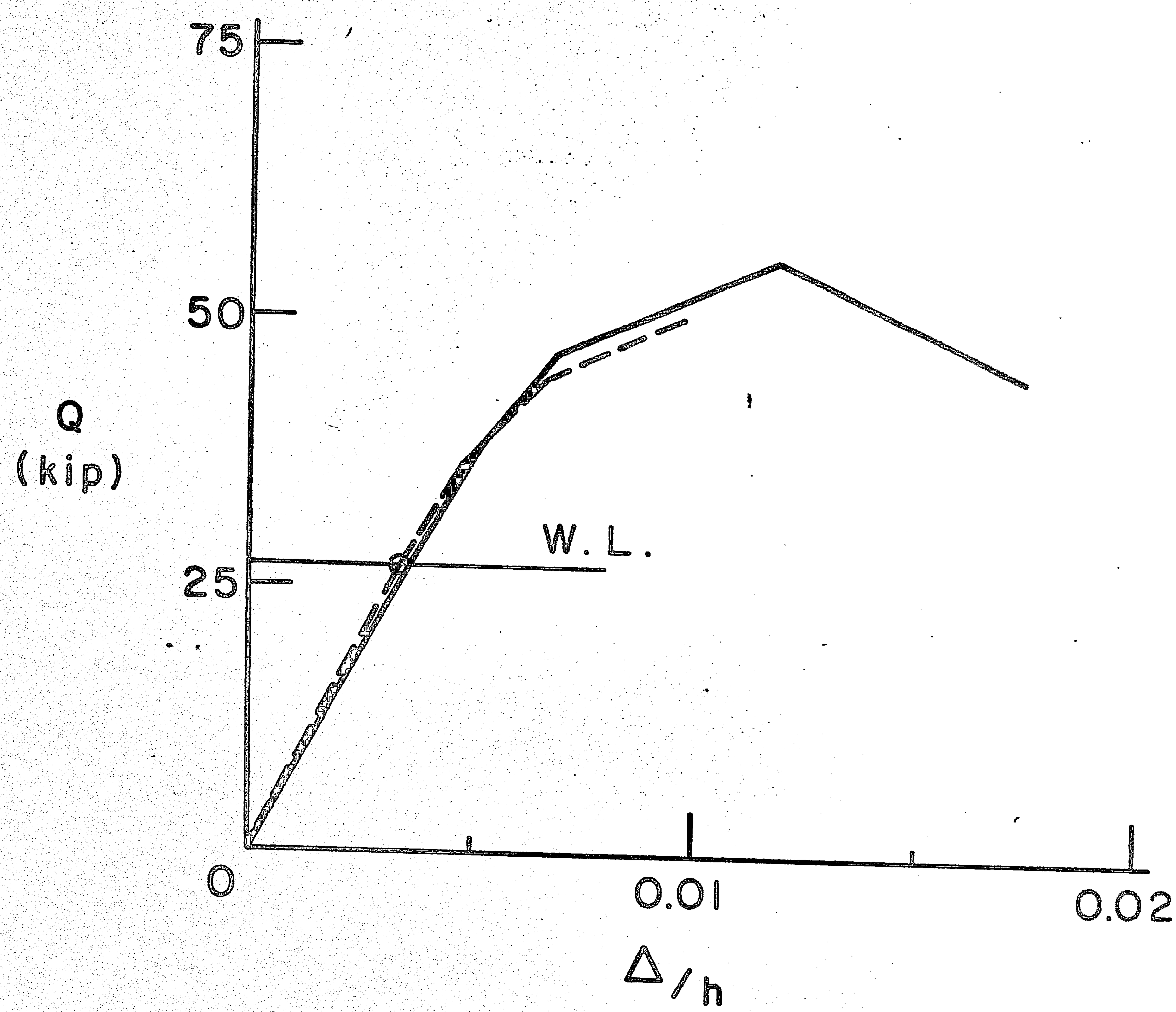

Fig. 4.8 Load-Deflection Curve of Level 6

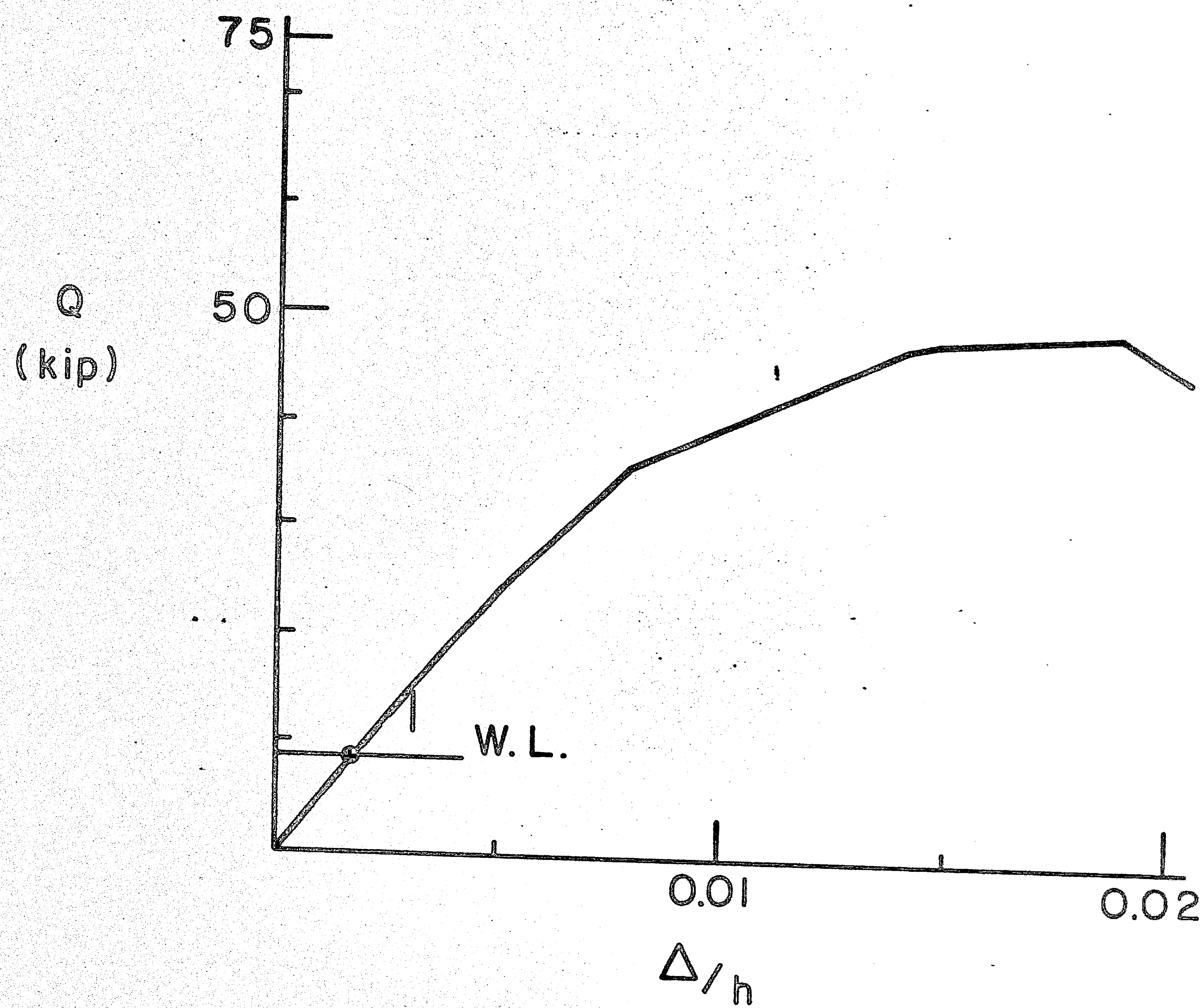

Fig. 4.9 Load-Deflection Curve of Level 2

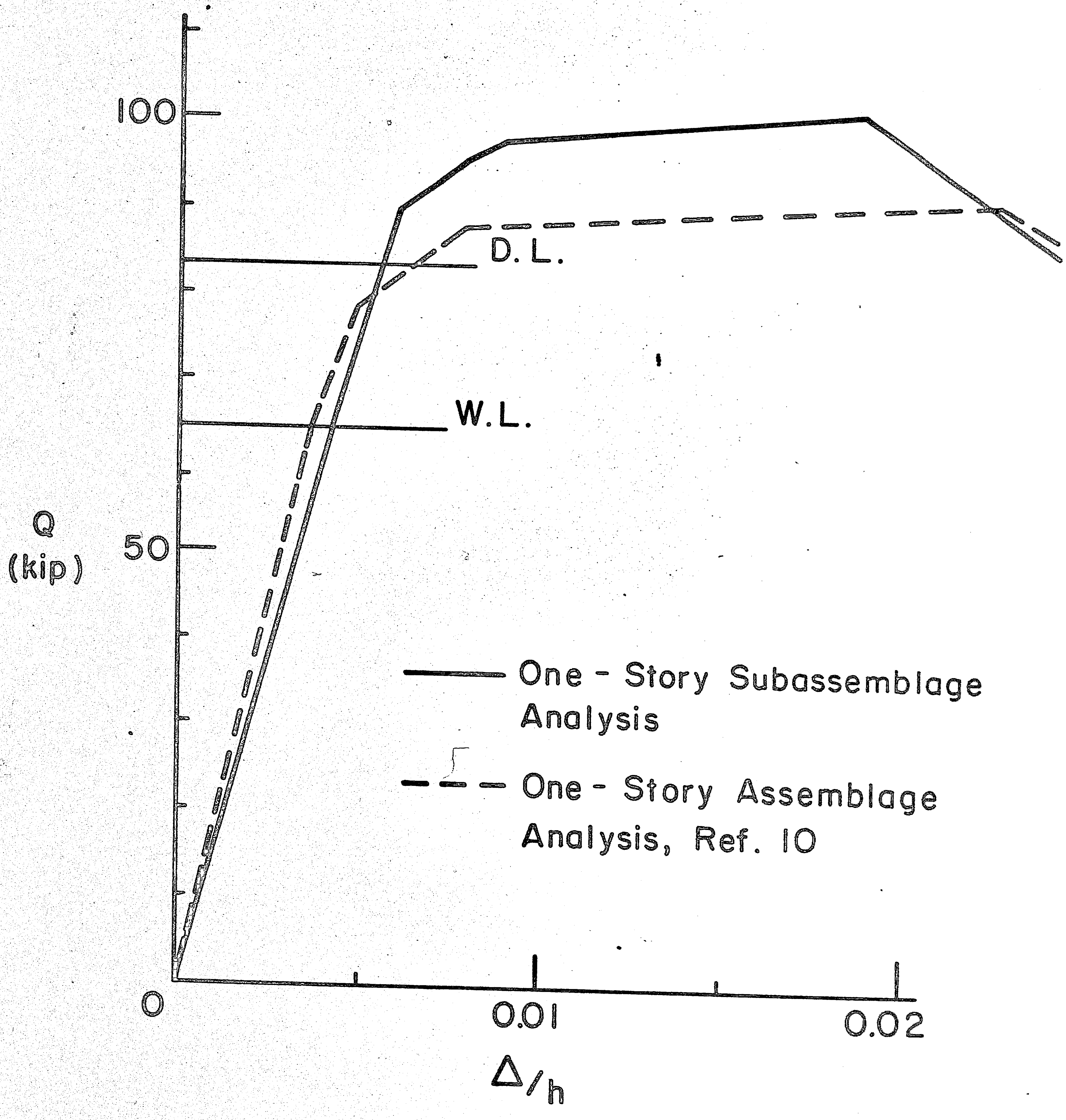

Fig. 4.10 Load-Deflection Curve of Level 14

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